Nonextensive effects on the phase structure of QHD

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We investigate nonextensive effects on phase transition in nuclear matter in the context Walecka many-body field theory. Changes in the general behaviour are observed when the results calculated for the nuclear matter at vanishing baryon density are compared to those obtained through the standard Fermi-Dirac distribution. It is observed a dependence between the nonextensive parameter q and the coupling constants C_S^2 of the phase transition. A numerical relation for this thermodynamical dependence is also proposed.

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I. INTRODUCTION

Non-extensive statistical mechanics (NESM) is a generalization of thermodynamics and statistical mechanics aiming to overcome a number of physical systems that possess exotic properties, such as broken ergodicity, strong correlation between elements, multi fractality of phase-space and long-range interactions. The NESM framework proposed by Tsallis [1] is based on the non-additive q-entropy

$$S_q = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1}, \tag{1}$$

where k is a positive constant, W is the number of microscopic states, and p_i is a normalized probability distribution.

In fact, additivity for two probabilistically independent subsystems A and B is generalized by the following pseudoadditivity:

$$\frac{S_q(A,B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)S_q(B)}{k^2}\ .$$

For subsystems that have special probability correlations, extensivity is not valid for Boltzmann-Gibbs (BG) entropy, but may occur for S_q with a particular value of the index $q \neq 1$, called the q-entropic parameter. In the limit $q \to 1$, BG entropy is additive, i.e,

$$S_{BG}(A,B) = S_{BG}(A) + S_{BG}(B)\ ,$$

and Eq. (1) reduces to the usual Boltzmann and Gibbs formula $S_1 = k\sum_{i=1}^{W} p_i \ln p_i$, which is the fundamental expression describing systems that usually do not present the exotic properties described above.

Several consequences of this generalized framework have been investigated in the literature [2] (see also [3] for a regularly updated bibliography). This includes systems of interest in high energy physics, e.g., the problem of solar neutrino [4, 5], the charm quark dynamics in a thermal quark-gluon plasma for the case of collisional equilibration [6], interpretations for central Au-Au collisions at RHIC energies in a Relativistic Diffusion Model (RDM) [7], among others [8, 9].

Recently, an interesting connection between quantum statistics and Tsallis NESM has been presented in Ref. [16]. The main result of this study is that, for $q > 1$, a q-generalized quantum distributions for fermions and bosons are given by

$$n_q(\mu,T) = \frac{1}{\tilde{e}_q(\beta(\epsilon-\mu)) \pm 1}, \tag{2}$$

where $\tilde{e}_q$ reads

$$\tilde{e}_q(x) = \begin{cases} [1+(q-1)x]^{\frac{1}{q-1}} & \text{if} \quad x > 0 \\ [1+(1-q)x]^{\frac{1}{1-q}} & \text{if} \quad x \leq 0\ . \end{cases} \tag{3}$$

and $x = \beta(\epsilon - \mu)$. In the $q \to 1$ limit, the standard Fermi-Dirac distribution, $n(\mu,T)$, is recovered. As physically expected, as $T \to 0$, $n_q(\mu,T) \to n(\mu,T)$. This amounts to saying that for studies of the interior of neutron stars (where, in nuclear scale, $T \simeq 0$) we do not expect any nonextensive signature. On the other hand, in heavy ions collision experiments or in the interior of protoneutron stars, with typical stellar temperatures of several tens of MeV (1 MeV= 1.1065×10^{10} K), nonextensive effects may appear.

In this paper, by following the arguments of Ref. [11], we discuss non-extensive effects on nuclear matter at zero baryon density. To this end we adopt the relativistic phenomenological approach developed by Walecka [10], the so-called quantum hadrodynamics (QHD). By taking into account the results on the phase transion in nuclear matter discussed in Ref. [15], we show that there exist

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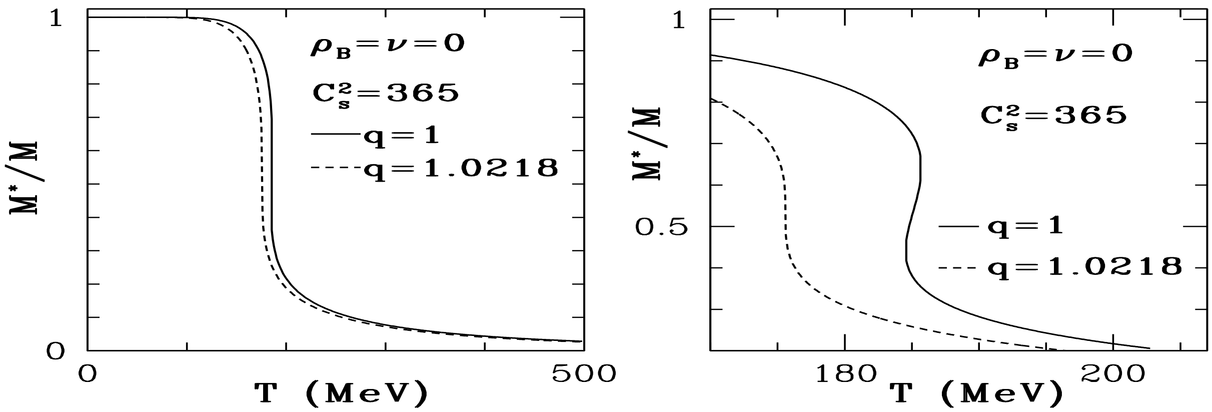

FIG. 1: The solution M^*/M of Eq. (5) for nuclear matter ($\gamma = 4$) at vanishing baryon density as a function of temperature for the same value of C_S^2 and two different values of the parameter q. The right panel shows the same results but with a stretched temperature region around the point of phase transition.

a thermodynamical dependence between the statistical correlations characterized by non-extensive parameter q and the coupling constant C_S^2. A numerical relation in the $q \times C_S^2$ plane resulting in regions of differents thermodynamical behaviors is also proposed.

II. NONEXTENSIVITY ON QHD

In the mean-field approach (for details see reference [10]), we have the scalar density

$$\varrho_S = \frac{\gamma_{\rm N}}{(2\pi)^3}\int \frac{M^*}{E^*(k)}[n_q(\nu,T)+n_q(-\nu,T)]d^3k \ , \quad (4)$$

where $E^*(k) = \sqrt{k^2 + M^{*2}}$, $\beta = 1/k_BT$, M^* is the effective mass

$$M^* = M - g_\sigma\sigma = M - \frac{g_\sigma^2}{m_\sigma^2}\rho_S \ , \quad (5)$$

and, instead of the usual Fermi-Dirac distribution, we have used for $q > 1$ the q-generalized quantum distribution for fermions, $n_q(\nu,T)$, given by Eq. (2).

The baryon number density, the energy density and pressure are given, respectively, by

$$\varrho_B = \frac{\gamma_{\rm N}}{(2\pi)^3}\int [n_q(\nu,T)-n_q(-\nu,T)]d^3k, \quad (6)$$

$$\begin{aligned}\varepsilon \ = \ & \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\varrho_B^2 + \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}(M-M^*)^2 + \\ & \frac{\gamma_{\rm N}}{(2\pi)^3}\int E^*(k)[n_q(\nu,T)+n_q(-\nu,T)]d^3k \ , \quad (7)\end{aligned}$$

$$\begin{aligned}p \ = \ & \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\varrho_B^2 - \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}(M-M^*)^2 + \\ & \frac{1}{3}\frac{\gamma_{\rm N}}{(2\pi)^3}\int \frac{k^2}{E^*(k)}[n_q(\nu,T)+n_q(-\nu,T)]d^3k \ , \quad (8)\end{aligned}$$

The parameter $\nu \equiv \mu - g_\omega\omega_0 = \mu - (g_\omega/m_\omega)^2\varrho_B$ is the effective chemical potential, and γ_N is the multiplicity factor ($\gamma_N = 2$ for pure neutron matter and $\gamma_N = 4$ for nuclear matter).

Since the function $\tilde{e}_q(x)$ is a *deformed* exponential, the mathematical convergence of the integrals in Eqs. (4)-(8) must be verified. In Ref. [11] it was pointed out that, to simultaneously satisfy the convergence of all integrals in Eqs.(4)-(8), we have found that $1 < q < 5/4$, consistent with the limits of q in the range $q \in (0, 2]$ of Ref. [17]. Additionally, we also use for the coupling constants the values of reference [10], namely[19],

$$\left(\frac{g_\sigma}{m_\sigma}\right)^2 = 11.798 \text{ fm}^2 \ \text{ and } \left(\frac{g_\omega}{m_\omega}\right)^2 = 8.653 \text{ fm}^2 \ , \quad (9)$$

which are fixed to give the bind energy $E_{\rm bind} = -15.75$ MeV and $k_F = 1.42$ fm^{-1}.

III. THE PHASE STRUCTURE AT ZERO BARYON DENSITY

In this Section, by considering the same arguments of Ref. [15], we explore the non-additive phase structure of the effective Lagrangian at vanishing chemical potential and baryon density ($\varrho_B = \nu = 0$). To this end, we first consider the range of values for the coupling constant C_S^2 given by

$$C_S^2 = (\frac{g_\sigma}{m_\sigma})^2 M^2 \quad (10)$$

in the coupling-constant plane shown in Fig. (1) of Ref. [15]. At vanishing chemical potential, the terms with the baryon density do not appear in Eqs. (7) and (8). In what follows, we will take as an example $C_S^2 = 365$.

Theis *et al.* [15] showed that the order of phase transition is strongly dependent on the actual value of the coupling constant C_S^2. However, in Sec. V of Ref. [11],

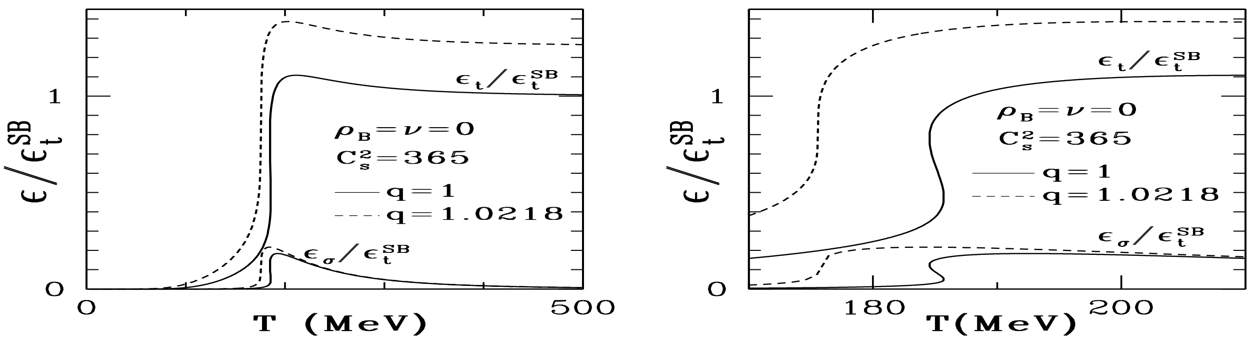

FIG. 2: The total energy density ϵ_t and the scalar-field energy density ϵ_σ divided by the $q = 1$ Stefan-Boltzmann limit ϵ_t^{SB} (given by Eq. (26) of Ref. [11]) as a function of temperature, at zero baryon density of nuclear matter ($\gamma = 4$). The same value of C_S^2 is considered for two different values of the parameter q. In the right panel, we show the same results in the stretched temperature region near the transition point.

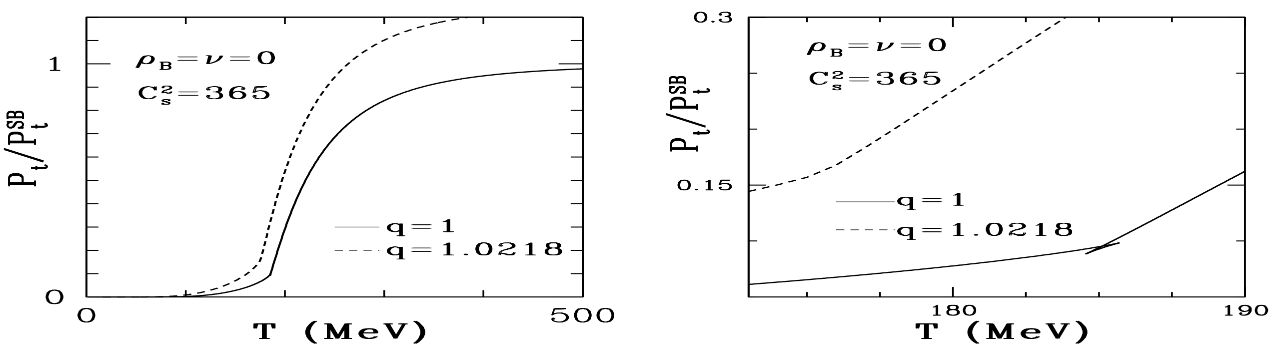

FIG. 3: The total pressure divided by the corresponding $q = 1$ Stefan-Bolztmann limit P_t^{SB} (Eq. (26) of Ref. [11]) as function of temperature. The right panel shows the first order phase transition point for q (solid curve) **in the stretched temperature range**. For $q = 1.0218$ (dotted curve) the phase transition is of second order.

it is shown that we can avoid Maxwell construction by the variation of the parameter q. Since the order of transition depends on C_S^2 and q, the natural question is whether there exists some relation between C_S^2 and q at zero baryon density.

In Fig. (1), the sudden drop in M^* around $T \sim 185$ MeV determines the abrupt rise of the energy density and pressure. We note that for $q = 1$ (Fermi-Dirac statistics) the self-consistency equation (5) has three solutions around $T \sim 185$ MeV imposing a sudden rise in the energy density and a peak in the specific heat. This behavior is shown in Figs. (2)-(4), where the temperature dependence of total energy density, pressure and specific heat divided by the corresponding high Stephan-Boltzmann temperature limit (for $q = 1$), given by Eq. (26) of Ref. [11], are displayed. The behavior of the curves for $q = 1$ characterizes a first order phase transition with the pressure curve crossing itself twice at $T \sim 185$ MeV. We observe that in this region the energy density and presure are also triple valued and that the specific heat is negative. The value of $q = 1.0218$ was obtained by requiring that energy density and pressure to be single valued characterizing a second order phase transition with non-negative specific heat. This allows to obtain a relation between C_S^2 and q explained below [20].

A. $C_S^2 - q$ relation

In order to obtain such a relation, in what follows we investigate the thermodynamical behavior of the nuclear matter for several values of the coupling constant C_S^2 in the $q \times C_S^2$ plane. For each value of C_S^2, the corresponding parameter q, for which the transition is of second order, is determined. Thus, a curve in the $q \times C_S^2$ plane is obtained as shown in Fig. (5a). Below this curve the phase transitions are of the first order and above it the thermodynamical behavior is smooth. Let us now show how the calculation is made via specific heat.

Differently from the treatment discussed in Ref. [15], the mathematical structure of the self-consistency equation in our approach is not simple, so that the calculation must be done numerically. We observe that the specific heat calculated from Eq. (7) is linear in dM^*/dT. Thus, whenever there is a sudden fall in M^*, we see a peak in the specific heat. By writing

$$C_H = \frac{d\varepsilon}{dT} = \frac{d\varepsilon}{dM^*}\frac{dM^*}{dT} \tag{11}$$

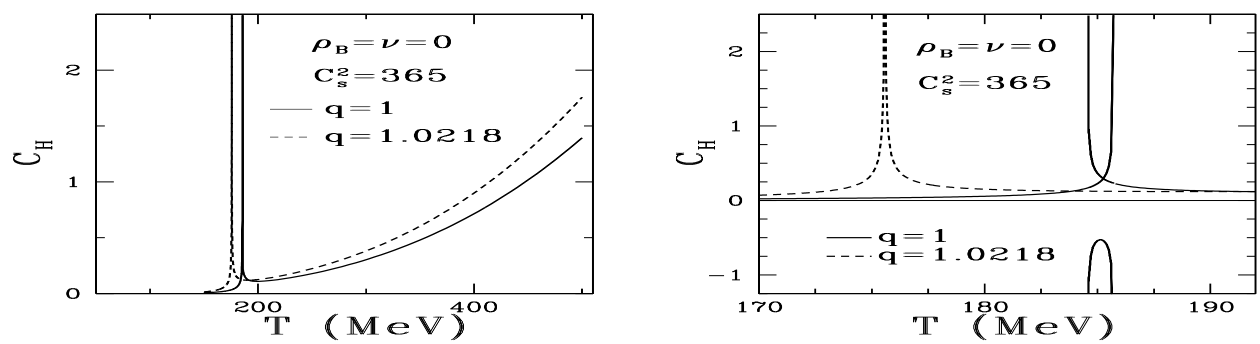

FIG. 4: The specific heat of nuclear matter ($\gamma = 4$) divided by the corresponding $q = 1$ Stefan-Boltzmann limit (Eq. (26) of Ref. [11]) as function of temperature for the same value of C_S^2 and two different values of the parameter q. In the right panel, the same results in the stretched region around the phase transition point. For $q = 1$ and $q = 1.0218$ the phase transitions are, respectively, of the first and second order.

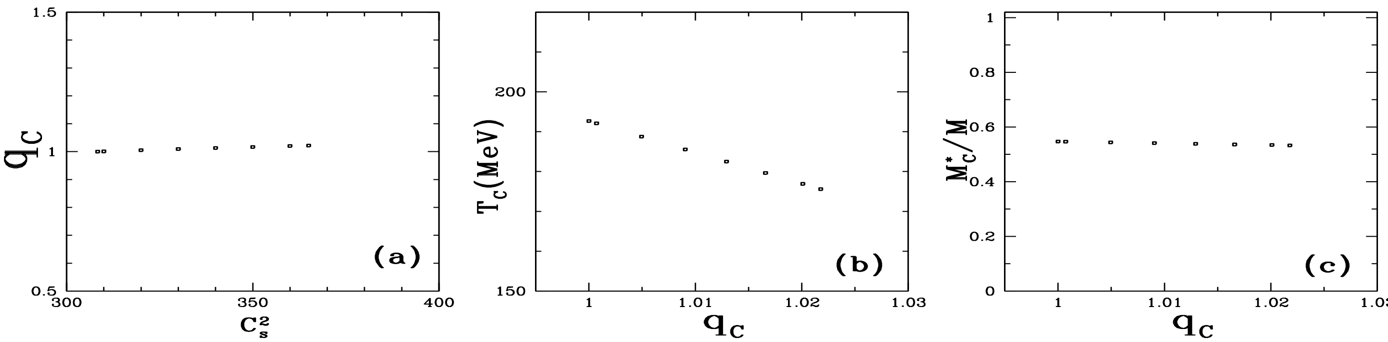

FIG. 5: Panel (a): Values of the critical parameter q for which the phase transitions are of second order as function of the coupling constant C_S^2, for nuclear matter ($\gamma = 4$) at zero baryon density. Panel (b): The temperature corresponding to Panel (a) as function of q_C. Panel (c): The same as in Panel (b) but for the effective mass.

we can see from Eq. (5) that

$$\frac{dM*}{dT} = \frac{-2C_{M^*}\frac{M^*}{T}\int_0^\infty \frac{2k^2+M^{*2}}{E^*(k)}\, n_q dk}{1 + 2C_{M^*}\int_0^\infty \frac{k^2-M^{*2}}{E^*(k)} n_q dk} \qquad (12)$$

where

$$C_{M^*} = (g_\sigma/m_\sigma)^2\gamma_N/\pi^2 \equiv C_S^2/M^2 .$$

The singularities of dM^*/dT lie in the curve determined by the vanishing of the denominator. Using Eq. (5), this condition becomes

$$M - 2C_{M^*}M^{*3}\int_0^\infty n_q\ \frac{dk}{E^*(k)} = 0\ . \qquad (13)$$

Note that, for $q = 1$, we fully recover Eq. (18) of Ref. [15].

The number of intersections of the solutions obtained from Eq. (13) and the self-consistency equation given by Eq. (5) determines how decoupling happens. We have a first or a second order phase transition, respectively, for two or one intersections. If there is no intersections, the thermodynamical behavior is continuous. The numerical results shown in Fig. (5a) can be summarized as follows:

- For q lying below the $q \times C_S^2$ curve the phase transitions are of the first order.
- For q lying on the $q \times C_S^2$ curve the phase transitions are of the second order. The corresponding values of temperatures and effective masses are shown in Figs. (5b) and (5c).
- For q lying above the $q \times C_S^2$ curve the decoupling is continuous.

Therefore, the order of phase transitions depends not only on the actual values of C_S^2 but also on the values of q.

IV. FINAL REMARKS

We have investigated some effects of the Tsallis NESM on the mean field theory of Walecka (QHD) [10]. Instead of the standard Fermi-Dirac nucleon and antinucleon distribution functions, we have used the q-quantum distribution obtained in Ref. [16]. Although there is no long range interactions in nuclear matter (Coulombian interaction is not included in our analysis), it is worth empha-

sizing that the NESM effects appear due to the strong statistical correlations between elements of nuclear system.

We have examined the phase structure of nuclear matter at high temperature and at zero baryon density. The effective Lagrangian of QHD theory is considered for the same set of values of the coupling constants in the coupling constant-plane of Ref. [15]. Given that, at vanishing baryon density, the term with the baryon density do not appear in the equation of state, a relation between the coupling constant C_S^2 and the parameter q is obtained. As discussed, such a relation determines, in the $q \times C_S^2$ plane, regions of different thermodynamical behaviors.

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[19] For the purpose of the present work, the values given in Eq. (9) suffices to investigate the effects of the nonextensivity in neutron and nuclear matter. Variations of the coupling constants, within the acceptable values given in current literature, do not qualitatively affect the conclusions.

[20] We remark that near $T \sim 180$ MeV, the effective mass being very small, the system decouples to an almost freemassles nucleon gas. In the present case, this depends on the actual values of C_s^2 and q. As pointed out in Ref.[15], this is analogous to the expected chiral phase transition in high-temperature QCD. However, in Walecka theory, there is no liberation of the internal constituents of the nucleons. Thus, the phase transition in QHD theory can not be interpreted as a baryon-quark matter one.